Worker protection and screening*

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Abstract

We develop a model where workers differ ex-ante in their ability to find good matches. Firms can screen the match quality in the hiring process, or learn about it during production. We uncover the presence of a screening externality: the firms' decision to screen and filter workers lowers the quality of the applicant pool and disincentivizes future job creation. We calibrate the model using administrative data and study the role of a worker protection policy that prevents firms from lay off bad matches during the production process. In the calibrated model, this policy has positive welfare effects if it lessens the presence of the screening externality in the economy.

JEL Codes: J60, J63, J65, J50

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1 Introduction

Under the presence of heterogeneous match productivity, firms may want screen and filter less productive matches. Pries and Rogerson (2005) study this channel and interaction between ex-post match heterogeneity and worker protection: workers are ex-ante homogeneous but find – by chance – matches that differ in their productivity. A worker protection policy that raises the cost of layoffs increases the risk associated with a bad hire and thereby disincentivizes job creation.

A recent literature has documented empirically the presence of worker ex-ante heterogeneity, and shown that it drastically changes macroeconomic dynamics (see for example Pries (2008), Ferraro (2018), Hall and Kudlyak (2019), and Gregory, Menzio, and Wiczer (2021)). We revisit the role of worker protection policy in a framework where workers do not only differ in their ex-post match heterogeneity, but also in their ex-ante ability to find good matches. In the model that we develop, the workers' ability to find good matches differs according to their permanent type: low-typed workers have a lower chance of finding a good match than high-typed workers[1]. Firms cannot target workers of a given type in their hiring process. Instead, they meet unemployed workers at random. We motivate these key ingredients by our subsequent empirical analysis: we assign workers into types using Danish administrative data following the methodology outlined by Gregory, Menzio, and Wiczer (2021). We find that 90% of Danish workers belong to the high-type cluster, which is characterized by an average unemployment rate of 5%. The remaining 10% have a much shorter average match duration and an unemployment rate of 22%. The model will make sense of these facts by assigning to the low-type cluster a significantly lower ability to find a good match. We then show that these low-type workers can be found across almost all occupations, industries, education groups and ages. This suggests that firms have a difficult time identifying a worker's type in the hiring process.

Our model combines the Diamond (1982)-Mortensen (1970)-Pissarides (1979) framework with ingredients from Pries and Rogerson (2005). We make two important departures: first, we allow for ex-ante heterogeneity as outlined previously. Second, we alter the firms' ability to screen during hiring to be consistent with ex-ante heterogeneity. As in Pries and Rogerson (2005), match productivity is both an inspection and an experience good: firms learn about the quality of the match during production and separate unproductive matches. During the hiring process, firms take into account the share of low-typed workers among the unemployed, as it governs the extent to which a random worker turns out to be a good match.

In this setup, we demonstrate how the relationship between ex-post heterogeneity and ex-ante heterogeneity can significantly alter macroeconomic dynamics. We uncover a "screening externality": when firms screen and select for good matches in the hiring process or during production, they increase the share of low-typed workers among the pool of unemployed. In the well-known search externality (Diamond, 1982), one participant's search activity reduces the likelihood of other participants to find match. Here, additionally, the decision of a vacancy to screen for good matches and reject bad matches reduces – in expectations – the quality of the pool of unemployed, reducing the value of opening a vacancy and thereby deterring future job creation. A worsening of the equality of the pool of unemployed, the ex-ante heterogeneity, in turn increases the incentives for screening – a vicious cycle.

[1]This is a simplified approach that generates a worker heterogeneity similar to Gregory, Menzio, and Wiczer (2021)

We first illustrate this mechanism analytically and show that the screening externality is stronger when the ex-ante heterogeneity is stronger. We then calibrate our model to the Danish economy and the worker types that we have identified therein. In our calibration, the model makes sense of these stark differences observed match differences by worker-type by assigning a much lower probability of finding a good match to low-type workers. The calibrated model thus features a relatively large screening share, and in equilibrium features a strong screening externality. We use the calibrated model to revisit the role of a worker protection: we introduce a policy where firms are no longer allowed to lay off bad matches during employment. Relative to Pries and Rogerson (2005) and the otherwise extensive theoretical literature on worker protection (Lazear, 1990; Bertola, 1990; Lindbeck, 1993; Hopenhayn and Rogerson, 1993; Lindbeck and Snower, 2001; Pries and Rogerson, 2005; Ben Zeev and Ifergane, 2022), our framework suggests a new role for worker protection: by reducing the aggregate screening in the economy, it improves the quality of the pool of the unemployed, and encourages job creation.

We find that the qualitative effect of worker protection on welfare depends significantly on a key parameter, σ_k. This parameter governs the dispersion of screening cost across firms, and thereby controls the extent to which aggregate screening responds to factors that affect the screening decision. When the worker protection policy takes place, firms can no longer rid themselves of bad matches during employment, and partly substitute towards screening during the hiring process. σ_k governs the extent to which this substitution takes place: when aggregate screening increases a lot in response to the worker protection policy, the quality of the unemployed pool deteriorates, deterring job openings and worsening unemployment and consumption. When instead aggregate screening increases less in response to the worker protection policy, it manages to improve the quality of the pool of the unemployed, reducing unemployment and improving aggregate consumption. We show in a heterogeneity analysis that the low-type workers are bearing the brunt of the impact of the worker protection policy: high-type workers are unaffected by the worker protection policy, and low-type workers primarily drive changes in the aggregate unemployment rate. This mirrors the finding that low-type workers are the main drivers behind the response of aggregate unemployment to aggregate TFP shocks (Pries, 2008; Ferraro, 2018; Gregory, Menzio, and Wiczer, 2021).

This paper emphasizes the role of combating the screening exernality but does not mean to say that this is the dominant channel through which worker protection affects the economy. We encourage future research that integrates other important channels (for example precautionary savings in a HANK environment) to make such quantitative statements and align the model with the empirical literature (for example, see Bertola (1990), Belot, Boone, and Van Ours (2007), Dal Bianco, Bruno, and Signorelli (2015), and Holmlund (2014).

Our research adds to a literature that highlights the role of screening and discrimination in the hiring process: Jarosch and Pilossoph (2019) study substitutability between screening and statistical discrimination, Acharya and Wee (2020) highlight the role of rational inattention in the screening process, and Masters (2014) show that statistical discrimination in the hiring process can be a vicious cycle.

From a modeling perspective, we complement recent papers that study the role of worker heterogeneity in macroeconomic dynamics. In some of these models, vacancies can direct their search towards workers of a specific type (in Ferraro (2018) and Gregory, Menzio, and Wiczer (2021) these types differ in worker productivity and their match productivity, respectively). These models are analytically tractable as firms can target workers of a specific type, and therefore

need not know and keep track of the distribution of types in the pool of unemployed. In such a setting, the screening externality cannot arise as the quality of the pool of unemployed is irrelevant for vacancies. Pries (2008) builds a random-model with two types of workers that differ in their productivity, but does not study the role of screening. Acharya and Wee (2020) study an information-based framework where the dynamics between screening and the quality pool of the unemployed stem from rational inattention.

The remainder of the paper is as follows. Section 2 develops the model and provides the characterization of the equilibrium. Section 3 applies the framework to the Danish labour market by estimating worker types using the administrative records. Section 4 studies the introduction of worker protection, and section 5 concludes.

2 Model

The purpose of the model is to study the impact of worker protection in an environment where matches differ ex-post in their quality, workers differ in their ex-ante likelihood of generating good matches, and firms can screen for match quality both during hiring and in the production stage. These two screening possibilities are substitutes: a worker protection policy that prevents firms from laying off bad workers during the production process will incentivize firms to screen more during the hiring process.

Figure 1 summarizes the building blocks of the continuous-time model. Unemployed workers u meet open vacancies v at random. At that point, firms can either decide to costly screen, which will inform them about the quality of the match with certainty. We will assume that bad matches are not viable, and only good matches will continue to production. Due to heterogeneous screening costs, some firms will decide not to screen in the hiring stage. These firms can decide to hire an applicant at random. This randomly hired applicant is a good match with probability P, which depends on the state of the economy. The quality of matches with randomly hired applicants is learned over time at rate $\hat{\delta}$. In the absence of worker protection, when a randomly hired applicant turns out to be a bad match, they are laid off. Both good and bad matches separate for reasons not modeled explicitly here which we summarize in the separation rate δ. We now describe these building blocks in more detail.

2.1 Worker heterogeneity and match quality

The quality of each match can be either *g*ood or *b*ad. We assume that workers either have a *l*ow or a *h*igh probability of finding a good match, and denote the corresponding probabilities of workers of each type as $\mathcal{P}_\ell$ and $\mathcal{P}_h$, with $\mathcal{P}_\ell < \mathcal{P}_h$[2]. The shares of low and high types of workers in the labor force are denoted by L_ℓ and L_h, respectively. We denote by e_i^g, e_i^b and u_i the mass of workers of type i that are either employed in good matches, employed in bad matches, or unemployed:

$$L_i = e_i^g + e_i^b + u_i$$

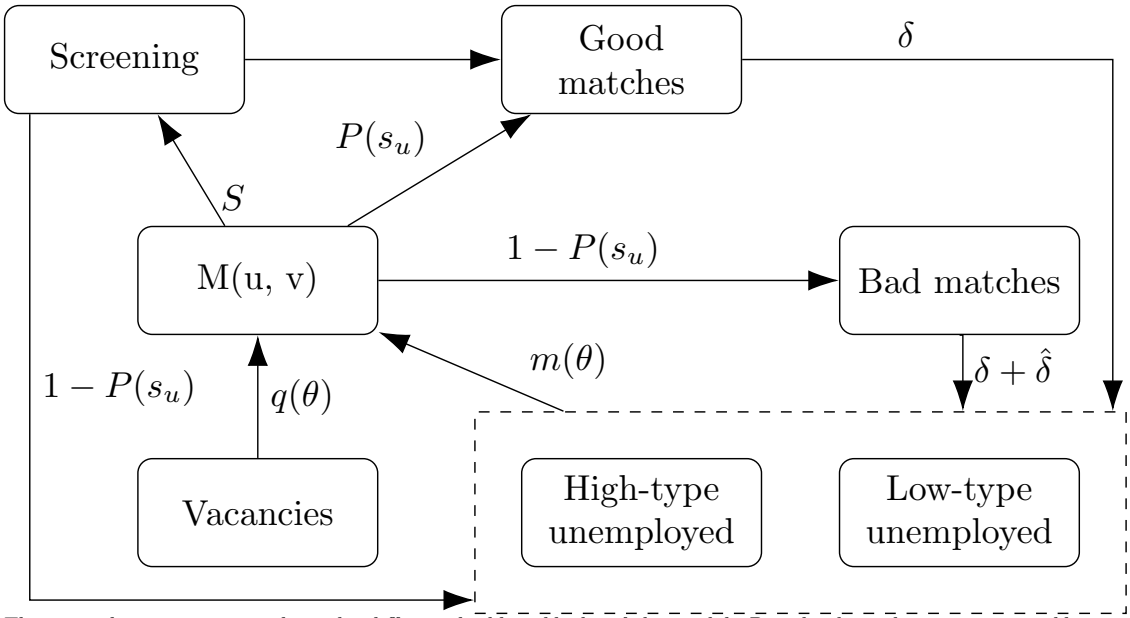
Ongoing matches of type i produce according to the match-specific component y^i. Firms pay a wage $w(t)$ that a combination of the productivity of good matches, and the worker's home production:

$$w(t) = \beta y^g + (1 - \beta)b.$$

For good matches, this wage could be retrieved as the result to a Nash Bargaining problem where the worker-firm pair does not produce for one period if they disagree on the wage. During this period, the firm generates no revenues, while the worker consumes the home production of the unemployed, b[3]. We assume that this wage is not only paid to good matches, but also matches where the quality is bad, or not yet revealed.

[2] Differences in worker types are similar to those in Gregory, Menzio, and Wiczer (2021), where low-type workers draw match-specific productivity from a more left-skewed distribution than high-type workers.

[3] We refer to e.g. Kaplan and Menzio (2014) for more information on this specific bargaining setup.



This provides an overview about the different building blocks of the model. Details about the various variables are provided in the main text.

Figure 1: Model overview

Here, we have in mind differences in worker-firm match quality in a narrowly defined labor market, i.e. conditional on occupation, industry, age, and many other characteristics. We will find empirically low-typed workers in Denmark conditional on these observable characteristics. We find it reasonable to entertain the assumption that, after controlling for these observable characteristics, wages do not (meaningfully) depend on match quality in such narrowly defined labor markets, especially as we will apply the model to Denmark with a unionization rate of around 75%[4].

This assumption is necessary since we – unlike Pries and Rogerson (2005) – want to study transition dynamics[5]. We believe that quality-dependent wages would not significantly alter our qualitative solutions: We solve in Appendix D a version of the model with quality-dependent wages and show that it generates steady states that are in line with our computational exercise in section 4.

We assume:

Assumption 1 *The surplus of bad matches is negative:*

$$w(t) > y^b.$$

This assumption simplifies the exposition as it allows us to ignore conditions under which a firm may want to keep a bad match [6].

[4]Union-bargained wages also serve as wage floors for many employers with non-unionized employees. While this still leaves room for individual negotiations, the assumption of quality-independent matches is not as strong as it would be for other less unionized countries.

[5]Looking ahead, the model will feature matches with unknown match quality. To be internally consistent, quality-dependent wages would imply that wages in matches with unknown quality are a function of their probability of being good. This probability will depend on state variables in the economy. When studying the response to shocks, we would have to keep track of a distribution of past hiring conditions under which matches had been formed.

[6]Note that Pries and Rogerson (2005) assume $y^b = b$ to ensure that bad matches are not profitable. In our context, due to the assumptions on quality-independent wages, we need not be as strict: there is room for $b > A^b$ under which bad matches are not profitable.

We can then write the value of ongoing good and bad matches to the employer as

$$\rho J^g(t) = y^g - w(t) - \delta J^g(t) + \dot{J}^g(t)$$
$$\rho J^b(t) = y^b - w(t) - \delta J^b(t) - \hat{\delta} J^b(t) + \dot{J}^b(t)$$

, where ρ and δ denote the discount rate and the separation rate. $\hat{\delta}$ is a parameter that relates to the rate at which employers learn about the match quality, and the ongoing worker protection policy. For good matches, the revelation of the type has no consequences. For bad matches, the revelation of the type leads to the termination of the match. When worker protection policies are in place, these terminations are no longer allowed. $\hat{\delta} = 0$ hence captures situations in which learning is not possible, or worker protection policies are in place. $\hat{\delta} > 0$ indicates a positive learning speed and an absence of worker protection.

2.2 Matching and screening

First, vacancies open and match with the unemployed according to a Cobb-Douglas matching-function $M(u, v) = Au^{\alpha}v^{1-\alpha}$. M is constant-returns-to-scale, and so we can write the rate at which workers meet vacancies, $m \equiv M(u, v)/u$, and the rate at which vacancies meet workers, $q \equiv M(u, v)/v$, as functions of the market tightness $\theta = v/u$ only. Upon matching, the firms draw a screening cost k from a log-normal random variable, whose cumulative density function is denoted as $G(k)$. They then decide to either screen, or to hire at random, if the expected value from doing so is larger than zero. Let P denote the probability of forming a good match at random. Low and high type workers differ in their probability of forming good matches, and so P depends on the share of low types among the pool of the unemployed, s_u:

$$P(s_u) = s_u(t)\mathcal{P}_\ell + (1 - s_u(t))\mathcal{P}_h$$
$$s_u(t) = \frac{u_\ell(t)}{u_\ell(t) + u_h(t)}.$$

The values of a screening and a non-screening vacancy are given by

$$\tilde{R}^S(k, t) = P(s_u(t))J^g(t) - k$$
$$R^N(t) = \max(P(s_u(t))J^g + (1 - P(s_u(t)))J^b(t), 0)$$

, where vacancies can refuse to hire at random if the expected value of doing so is negative.

A firm with a screening cost of k decides to screen iff $\tilde{R}^S(k, t) \geq R^N(t)$. The returns to screening strictly decrease in the screening cost k. The screening decision can therefore be characterized by a cutoff $\overline{k}(t)$: firms decide to screen if they draw a $k \leq \overline{k}(t) = -(1 - P(s_u(t)))J^b(t)$. The probability of drawing a screening cost below this threshold is given by

$$S(t) = G\left(-(1 - P(s_u(t))J^b\right), \tag{1}$$

and we denote the average value of a screening vacancy as

$$R^s(t) = \int_{-\infty}^{\bar{k}(t)} R^s(k,t)dG(k).$$

With this machinery set up, we can now define the value of opening a vacancy V as

$$V(t) = -c + q(\theta(t)) \left(S(t)R^S(t) + (1 - S(t)R^N(t)\right) + \dot{V}(t).$$

In equilibrium, the market tightness θ is governed by free entry: $V = 0$.

2.3 Laws of motion

The rate at which workers of type i find a job with a good or a bad match value are then given by

$$\begin{aligned} f_i^g(t) &= \mathcal{P}_i m(\theta(t)) \\ f_i^b(t) &= \begin{cases} (1-\mathcal{P}_i)m(\theta(t))(1-S(t)) & \text{if } V^N(t) \geq 0 \\ 0 & \text{else} \end{cases}. \end{aligned}$$

Both types of workers find matches at the rate $m(t)$, which turns out to be of good value with probability $\mathcal{P}_i$. With probability $1 - \mathcal{P}_i$, the batch is of bad value. In this case, they will only become employed if the vacancy chooses not to screen, and the value of hiring at random is non-negative.

This allows us to write the laws of motion for the stocks of labor as

$$\begin{aligned} \dot{e}_i^g(t) &= f_i^g(t)u_i(t) - \delta e_i^g(t) \\ \dot{e}_i^b(t) &= f_i^b(t)u_i(t) - (\delta + \hat{\delta})e_i^b(t). \end{aligned}$$

2.4 Steady state

An equilibrium of this economy is characterized by $\{S(t), \theta(t), \{e_i^g(t), e_i^b(t)\}_{i\in\ell,h}\}$ where the screening share $S(t)$ satisfies (1), $\theta(t)$ ensures that $V(t) = 0$, and the $\{e_i^g(t), e_i^b(t)\}$ are in line with the laws of motion of the economy.

We refer to steady state values by dropping the time index t. In steady state, θ, S, the job-finding rates, and the employment rates of each worker type are constant. The employment rates of both types $i \in \{\ell, h\}$ satisfy

$$e_i^g = \frac{f_i^g}{f_i^g + \delta}(L_i - e_i^b) \tag{2}$$

$$e_i^b = \frac{f_i^b}{f_i^b + \delta}(L_i - e_i^g). \tag{3}$$

The steady state unemployment mass of type i is given by

$$u_i = L_i \frac{1}{1 + \frac{1}{\delta}P_i m(\theta) + \frac{1}{\delta+\hat{\delta}}(1-P_i)(1-S)m(\theta)},$$

where the denominator is a combination of the different match types, their finding and separation rates. In the limiting case where $S = 1$ and $P_i = 1$, the job-finding rate f becomes equal to $m(\theta)$, and the unemployment rate of type i collapses to the familiar $\delta/(\delta + f)$ When instead $S = 0$ and $P_i = 0$, all matches are bad, and the unemployment rate of type i is given by $(\delta + \hat{\delta})/(\delta + \hat{\delta} + f)$, where once again $f = m(\theta)$.

2.5 The screening externality

As discussed before, the share of screening vacancies S depends on the likelihood of a good match, $P(s_u)$, and J_b, the value of a bad match. Recall that bad matches are not viable: $J_b < 0$. The following propositions characterize the relationship between the screening decision and the aggregate conditions of the economy.

Proposition 1

$$\frac{\partial S(t)}{\partial s_u(t)} = -G'(\overline{k}(t)(\mathcal{P}_h - \mathcal{P}_\ell)J^b > 0$$
$$\frac{\partial S(t)}{\partial J^b(t)} = -G'(\overline{k}(t))(1 - P(s_u(t)) < 0$$

The share of screening vacancies S increases in the share of low-type workers in the economy and decreases in the value of bad matches.

Proposition 2

$$\frac{\partial s_u}{\partial S} > 0$$

In steady state, an increase in screening raises the unemployment rate of both types. This effect is stronger for the low-type workers. Consequently, screening increases the share of low-type workers among the unemployed.

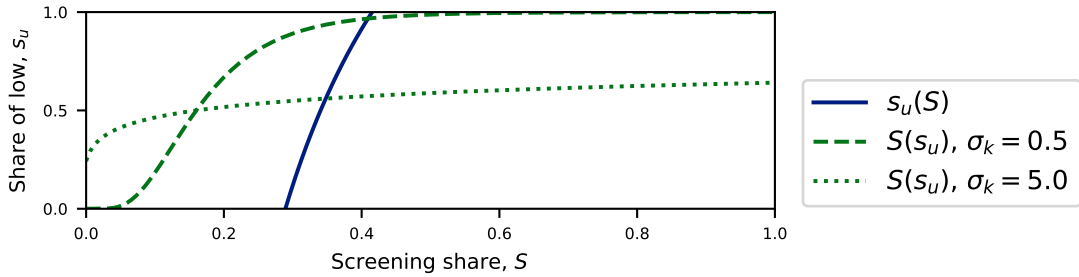
We specified Proposition 2 for the steady state where it is easy to prove, but it is likely true for many initial conditions of $\{u_i(t)\}$ that have arisen as a result of the laws of motion (2)-(3).

Screening externality In this environment, a firm's decision to screen depresses the rate at which vacancies are opened, as it lowers the quality of the pool of the unemployed. The negative impact on the pool of the unemployed is given by Proposition 2. Note that an increase in s_u lowers the odds of good matches in the hiring process, depressing V^N and V^S alike. In response, the market tightness θ has to fall to keep the value of opening a vacancy V at zero.

This screening externality is a vicious cycle: Proposition 1 implies that firms respond to an increase in s_u by increasing their screening efforts, which leads to a further increase in s_u.

From Proposition 1 it is clear that this latter effect increases in the difference $\mathcal{P}_h - \mathcal{P}_\ell$: the stronger the worker heterogeneity, the larger the screening externality.

The importance of the cost dispersion Figure 2 visualizes both $S = S(s_u)$ and $s_u = s_u(S)$ for the calibrated model and two values for the dispersion of the screening cost, σ_k. In line with the propositions, $S = S(s_u)$ and $s_u = s_u(S)$ are both strictly increasing functions. As expected,



We visualize the relationship between the screening share of vacancies S and the share of low types among the unemployed, s_u. $s_u(S)$ characterizes the effect of screening on the steady-state value of s_u. $S(s_u)$ characterizes the effect of s_u on the share of screening vacancies. We visualize the latter for two values of σ_k to emphasize the importance of the dispersion in screening costs on the sensitivity of aggregate screening with respect to changes in the fundamental variables that it is based upon.

Figure 2: Screening and the quality of the unemployed

the screening cost parameter turns out to be a key parameter: when the dispersion σ_k is high, S responds much less to changes in the fundamentals that drive the screening decision. For $\sigma_k = 0.5$, when s_u raises to 0.6, all vacancies screen their applicants. For $\sigma_k = 5$, 40% of vacancies will not screen their applicant even if the share of low-typed workers among the unemployed is 1.

The parameter σ_k is a key parameter that governs the extent to which S responds to s_u, and thereby, the strength of the screening externality. Its importance, together with the difficulty of finding reliable data to calibrate the *elasticity* of aggregate screening with respect to changes in the fundamentals, cautions us against choosing a single parameter value for our numerical simulation. Instead, we will provide calibrations for a set of values for σ_k, and discuss the extent to which our results depend on this parameter.

3 Application to the Danish labor market

In this section we apply our framework to the Danish economy. We first identify and describe the high and low-type workers using Danish register data. We then calibrate our model to the economy. Finally, we study the introduction of worker protection in this economy.

3.1 Data

Our main dataset is the BFL, a dataset that contains the universe of Danish wage payments since 2008. The BFL contains individual payments from an employer to an employee, together with the worker's occupation and their hours worked. We use the date of each wage payment to identify the beginning and the end of each employer-employee match more precisely than with most conventional tax-based employer-employee datasets[7]. This will be important as our approach to clustering workers into high and low types relies, among other moments, on the employmemt spell distribution.

We support the BFL with the DREAM data that contains the Danish social security benefits. The unemployment benefit information in DREAM allow us to identify whether a worker has received *any* unemployment benefits in a given month[8] In the data, we identify unemployed workers as those that do receive unemployment benefits and no wage income. Employed workers are workers that receive wage payments. Our model does not feature transitions between jobs or out of the labour force, and we thus only use transitions between these two states to compute job-finding and separation rates.

3.2 High-type and low-type workers in Denmark

We estimate the two types of workers by applying the methodology of Gregory, Menzio, and Wiczer (2021) to the Danish administrative data.

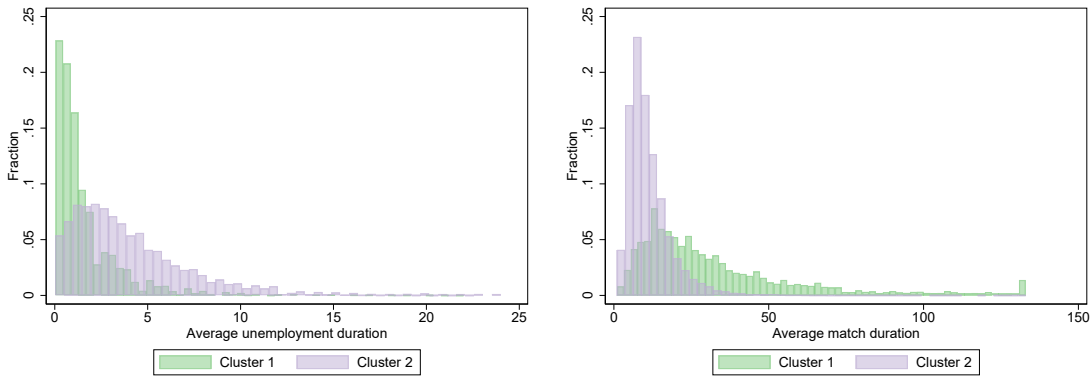
For this, we characterize the duration of each employment and unemployment spell of each worker into nine moments. The remaining moments capture the average number of jobs per unit of time, and the ratio of employed to non-employed time in the data. We apply a method of moments estimator to place workers into two clusters that minimizes the difference in the moments within each. Figure 3 plots the distribution of moments by worker type: high-type workers have significantly fewer jobs per unit of time and less time unemployed. Their employment spells are longer, and their unemployment spells are shorter.

The clustering approach is quite robust: the error rate in an out-of-sample prediction exercise is at 10%. About 10% of our workers are identified as low-type workers.

We discuss the distribution of the two types across industries, occupations, education groups and commuting zones in Appendix B. While there are some differences – most notably by education group – in the share of low types across the various dimensions, we find a significant and very similar share of low-type workers among almost all observable characteristics. This is in line with two of our modeling assumptions. First, it is difficult for the employer to identify a worker's type

[7] One disadvantage when relying on these dates to identify match duration is that some payments may occur after the match has ended. We remove such payments by relying on the associated hours

[8] Unfortunately, DREAM only contains a marker on whether any benefits have been received, and not the amount itself. Therefore, many part-time employed workers that still qualify for unemployment benefits will be marked as unemployed.



We compute the average duration of each employment and unemployment spell for each worker in our Danish sample. We then plot the distribution of workers across these averages rate by cluster.

Figure 3: Employment and unemployment spell distribution

Table 1: Labor market moments by type

Moment	High type	Low type
Share in labor force	0.879	0.121
Unemployment rate (combined)	0.049	0.224
Unemployment rate (DREAM)	0.248	0.446
Unemployment rate (BFL)	0.071	0.283
Job-finding rate (combined)	0.405	0.310
Separation rate (combined)	0.010	0.048

We compute labor market moments for the two estimated worker types. The unemployment rate is computed according to three definitions: DREAM uses only information on unemployment benefits. BFL only uses information on employment. The combined definition characterizes someone as unemployed if they receive unemployment benefits, but no wage income.

– and thereby the probability of a good match – purely by relying on observable characteristics. Second, by extension, it is difficult for the employer to target workers of a specific type in the hiring process. Consequently, this validates our approach as modeling the search process as random, rather then directed.

Before using the estimated clusters to calibrate the model, we will give a short summary of the labor market characteristics of both types. Table 1 summarizes the two types: the vast majority – 88% of the workers are assigned to the high-type cluster. This table also summarizes the unemployment rate of each cluster according to three definitions. For our calibration, we will use the COMB definition, which combines information from both DREAM and BFL. This definition only counts workers as unemployed if they have receive unemployment benefits and have no labour income. It is reassuring that the average of the COMB unemployment rate across both clusters, 6.9%, is in line with the average unemployment rate reported by Statistics Denmark (6.2%) for the same period. We also report the unemployment rate based on two other widely used measurements, the employer-employee data (BFL), and the unemployment benefits (DREAM) only. The former measure captures the non-employment rate. The latter measure characterizes part-time workers that receive supplementary benefits as unemployed. Either of these two measures yields a significantly higher unemployment rate than our preferred measure, highlighting the importance of combining the information from both data sets to obtain an accurate estimate. Finally, we also provide the job-finding rates and job-separation rates according to our preferred measurement of unemployment. Our model does not feature job-to-job transitions, and we consequently remove

Table 2: Parameters

Parameter	Value	Description	Source
General			
c	0.720	Vacancy cost	Calibration
δ	0.022	Separation rate	Calibration
ρ	0.003	Discount rate	Set exogeneously
b	0.975	Home production	Calibration
β	0.494	Bargaining weight	Calibration
$\mathcal{A}$	0.688	Matching function productivity	Calibration
α	0.720	Matching function elasticity	Shimer (2005)
Screening			
μ_k	-4.310	Screening cost mean	Calibration
σ_k	1.000	Screening cost dispersion	
$\hat{\delta}$	0.104	Learning rate	Calibration
Types			
P_ℓ	0.149	Good match prob., low type	Calibration
P_h	0.996	Good match prob., high type	Calibration
L_ℓ	0.121	Population share, low types	Own calculation
A_b	0.975	Productivity, bad match	Set exogeneously
A_g	1.000	Productivity, good match	Set exogeneously

The parameters of the calibrated model. As discussed in the main text, we first choose a set of parameters following our own empirical measurements or the literature. We then calibrate the remaining parameters to a set of moments outlined in table 3.

any job-to-job transitions for the estimation of our separation rates. The two types of workers do not differ significantly in their job-finding rates: high-type workers find a new job approximately 30% faster. Their main quantitative difference stems from the separation rate, as low-type workers lose their jobs almost five times as fast as high-type workers. Looking ahead to the calibration, the calibration will make sense of these empirical moments by setting a significant difference in workers of the two types in finding good matches, and a relatively high rate at which bad matches are identified and destroyed.

3.3 Calibration

We calibrate the model to the Danish economy and assume the period length to be one month. Figure 2 lists the parameters of the model. These can be grouped as follows. First, a set of parameters can be set exogeneously: these are either not very consequential, or can be observed directly. Second, a set of parameters govern the differential separation rates and job-finding rates between the two types of workers. These parameters only govern *differences* between the two types and hence can be calibrated independently from the absolute level of job-finding rates. A third set of parameters govern the absolute level of job-finding rates, and the elasticity of the matching surplus to productivity shocks. In a third step, we calibrate these parameters by adopting the Hagedorn and Manovskii (2008) procedure to this economy. Table 3 provides an overview over the moments used in the second and the third step. We now describe the calibration in detail.

Exogeneously set parameters We set the discount rate ρ in line with an annual discount rate of three percent. We set elasticity the matching function parameters α to 0.72, following Shimer (2005). It is likely that in reality, match productivity can take many different values, and that our data-generated moments are consistent with that. Yet, we assume for computational

Table 3: Moments

Parameter	Target	Value	Description
Matching			
$\eta_{w,A}$	0.500	0.500	Elasticity of wages w.r.t productivity
θ	0.200	0.200	Market tightness
c	0.720	0.720	Vacancy flow cost
u	0.070	0.071	Unemployment rate
Types			
s_u	0.386	0.382	Low types among the unemployed
S	0.803	0.803	Share of screening vacancies
$D^h(2)$	0.015	0.023	Separation rate month 1, high type
$D^\ell(2)$	0.063	0.075	Separation rate month 1, low type
$s_u f_\ell + (1 - s_u) f_h$	0.368	0.323	Average job-finding rate

The targets and moments of the calibrated model.

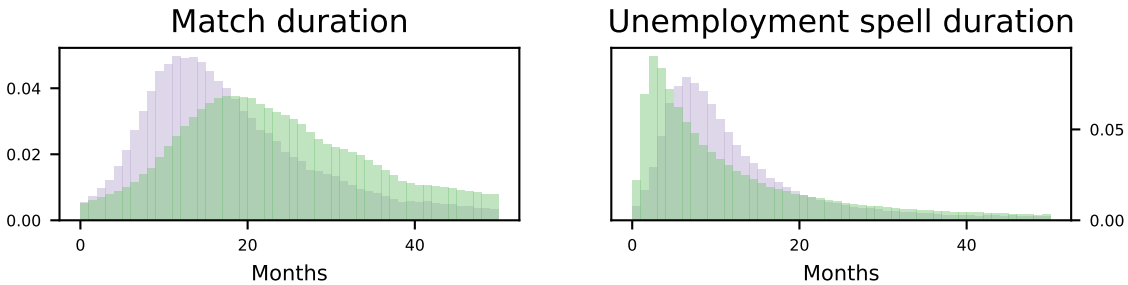
simplicity that there are only two values, y^b and y^g, and have to decide how to approximate this richer distribution with our two values. We set $y^g = 1$, which implies that our wages – which do not depend on the productivity of bad matches – remain comparable to the literature. Following Pries and Rogerson (2005), we choose $y_b = b$: workers in bad matches are as productive as in their outside option.

Finally, we have seen in the previous section that σ_k is a key parameter: it governs the sensitivity of screening. For the baseline calibration, we set $\sigma_k = 1$. However, one of our key findings is the importance of σ_k in determining aggregate outcomes: we will vary σ_k for our experiments and recalibrate the model for each value of σ_k.

Step 2: type-related parameters In the second step, we target a set of moments regarding the differential job-finding rates and separation rates by worker type. The first moment is the aforementioned ratio of the estimated job-finding rates, f_h/f_ℓ. Through the lens of our model, the separation rates for workers with low tenure are informative about the share of matches with low quality. Therefore, we choose two separation rates of workers with low tenure as our last moments. If we observed employment duration with great detail, we would choose $D^i(1)$. Unfortunately, we face a time aggregation issue: we observe employment at the calendar month level and therefore severely under count workers with a tenure of a single month. Therefore, we fall back to using $D^i(2)^i$ as our last two moments.

We use these moments to discipline the separation rate and learning rate $\delta, \hat{\delta}$, and the matching probabilities $\mathcal{P}_\ell, \mathcal{P}_h$. In this step, we also compute a screening share target $\hat{S}$. This is not a parameter, but another moment that we will use it in the next step to calibrate the screening cost parameter μ_k.

Step 3: matching-related parameters Finally, we calibrate the remainder of the matching parameters by applying the calibration procedure outlined in Hagedorn and Manovskii (2008) to our environment. We first structurally estimate the vacancy flow cost c. We then calibrate bargaining power β, home production b, matching function productivity A, and the screening cost parameter μ_k to match our estimate of the flow cost, the elasticity of wages with respect to productivity, the market tightness, the unemployment rate, and the previously estimated screening



Model-generated match durations and unemployment spell durations by type. Green: high-type workers. Purple: Low-type workers.

Figure 4: Average worker spell length in the model

share target $\hat{S}$.

We estimate the flow cost c following the stylized model introduced by Hagedorn and Manovskii (2008) that characterizes the vacancy cost as a sum of capital and labor costs. Denmark has similar a similar capital-to-labor ratio and labor share of income as the United States and so we use their estimate of the capital cost of hiring, 47.4% of monthly production. For the labor cost, we also refer to Silva and Toledo (2009) who list a hiring cost of 12% of monthly labor productivity. The estimated hiring cost includes screening costs. Therefore, we remove the average screening costs of the screeners $S \cdot E[k|k < \overline{k}]$ from hiring costs before computing our value of the flow cost.

Hoeck (2022) and Duque, Ramos, and Suriñach (2006) report Danish wage elasticities of around 0.05 and 0.8, respectively: we decide to target an intermediate value of 0.5. For this exercise, we decide to target an intermediate value of 0.5. For the unemployment rate, we target 0.07 – the average unemployment rate that we observe in our data according to the COMB definition. Ellermann-Aarslev (2018) reports a market tightness between 0.18 and 0.22, and we set $\theta = 0.2$ as our calibration target[9].

We have simulated one million households in the calibrated model for a length of 12 years, in line with the data. 4 displays the distribution of employment and unemployment spell lengths in the model, where we have simulated one million of households. It is reassuring that these spell distributions are qualitatively in line with their empirical counterparts in Figure 3.

[9]In line with this estimate, Hoeck (2022) reports an occupation-level market tightness of around 0.17 with a standard deviation of 0.1

4 Introduction of worker protection

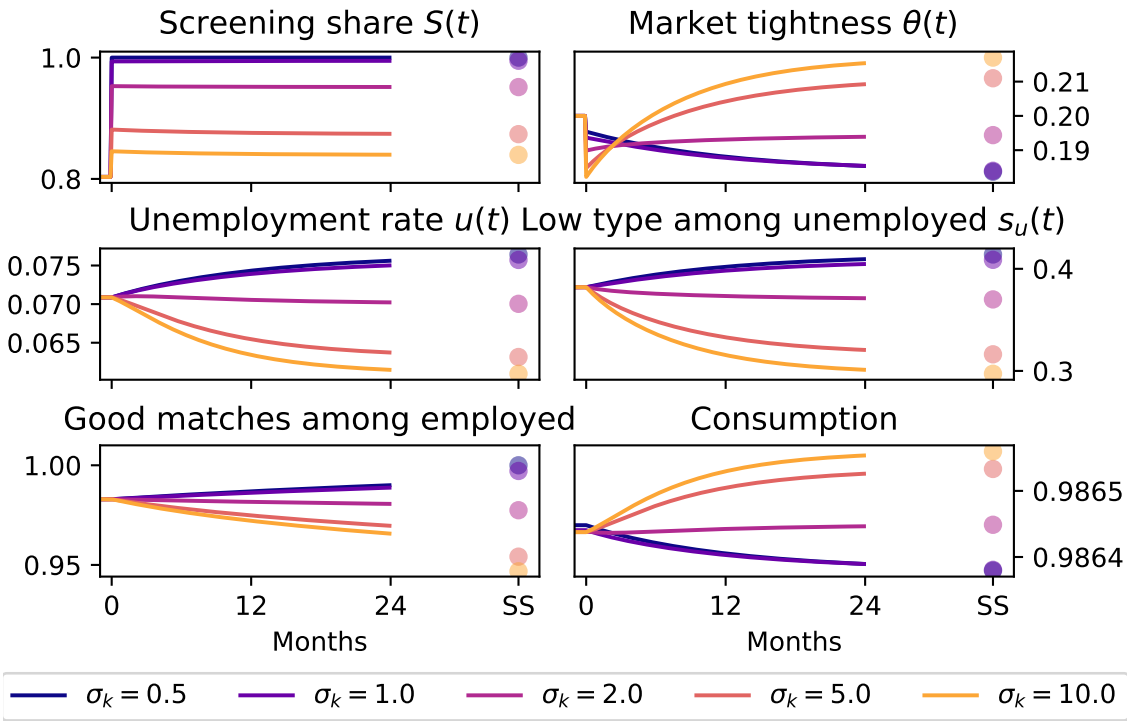
In this section we study the introduction of worker protection in our environment. To this end, we take the calibrated economy and introduce an unexpected change in the rate at which bad matches may be destroyed: we lower $\hat{\delta}$ from its previous value to 0. Technically, $\hat{\delta}$ is the rate at which the quality of unknown matches is revealed, which in other models might also affect wages. Because we assumed that wages are quality independent, setting $\hat{\delta} = 0$ is identical to assuming that bad matches no longer can be destroyed[10].

Figure 5 shows the evolution of the economy in response to this unanticipated policy change for different values of σ_k. The transition of some equilibrium variables to the new steady state is quite slow. For better exposition, the lines display the transition of various equilibrium outcomes throughout the first two years in response to the policy change, and the dots visualize the steady state value that they will eventually converge to.

We first consider the baseline value of $\sigma_k = 1$. The policy worsens the value of J^b, as bad matches now persist for a longer period. In line with Proposition 1, the share of screening vacancies increases. In fact, the value of J_b falls so much that the value of hiring at random becomes negative: firms with too high screening costs prefer to not hire at all. In consequence, the market tightness θ decreases and the job-finding rate of workers falls. Simultaneously, bad matches now separate at the much lower rate δ, and persist longer and the equilibrium effect of these two forces offsets: the unemployment rate remains roughly constant in response to the policy change. This however hides the heterogeneous effect of the policy change on workers from the low and the high type. Panel 4 shows that the share of low type among the unemployed, s_u, falls in the short-run. This is the direct effect of worker protection: low-type workers are more likely to be in bad matches that no longer can be destroyed when the quality is revealed. However, the raise in screening and the end of random hiring affects low-type workers dis-proportionally: in the long-run, their share among the unemployed increases from 17.5% to 18%. Panel 5 displays that as bad matches are no longer being hired, the share of good matches among the employed inevitably converges to one. Finally, panel 6 displays the consumption losses stemming from the policy. Most of these consumption losses do not stem from the immediate enactment of the policy but rather aggravate over time as the unemployment rate increases. Consumption losses in this model are relatively minor as we consider the worker's outside option b as part of consumption. Since b is calibrated to be relatively high, the fall in consumption associated with a worker's transition from employment to unemployment is relatively minor. A second reason for the relatively minor fall in consumption is that the quality of the matches is improving, which is associated with productivity gains.

Comparing the impact of the policy under different scenarios for σ_k is insightful for understanding the role of screening. A lower σ_k implies a smaller dispersion of the screening cost and more vacancies at the margin of screening. Consequently, the responsiveness of screening in the economy is higher. A higher responsiveness of screening implies that the immediate fall of market tightness is smaller, as firms can substitute discretionary layoffs more easily with screening during the hiring process. However, in cases where screening responds more the quality of the pool of unemployed, s_u, falls, deterring job creation. When screening responds less, s_u actually improves over time, and leading to increased job openings. In these cases, market tightness actually outperforms its

[10] We will discuss an alternative implementation of worker protection, the introduction of layoff costs, at the end of this section.



The response of the economy to the unexpected introduction of worker protection. Different colors correspond to calibrations of the model with a different screening cost dispersion, σ_k. Due to the slow transition to the new steady state, we only follow the economies for the first two years, and indicate the corresponding new steady states with circles.

Figure 5: Introduction of worker protection

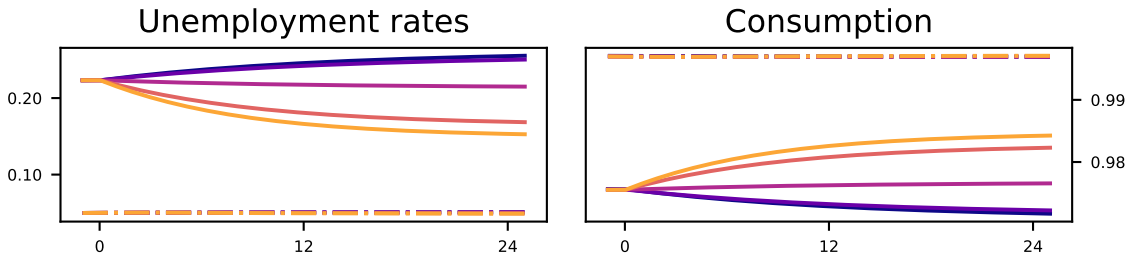
steadystate value: the unemployment rate falls as a consequence of worker protection. Instead, for small values of σ_k, the unemployment rate rises in the long run.

This highlights the importance of the screening externality: the introduction of worker protection prevents screening during the production process, and firms respond by attempting to screen during the hiring process. If the aggregate screening response is strong enough, this will outdo the extent to which the policy directly protects low-type workers. The extent to which screening responds to the protection policy governs whether the protection policy improves or worsens aggregate unemployment.

Panel 5 highlights the share of good matches among the employed. Indeed, scenarios with a small σ_k increase the share of good matches among the employed. Yet, the importance of the screening externality – and its deterring effect on hiring – outperforms quantitatively this positive effect of the increase in screening.

For completeness, Panel 6 displays the course of aggregate consumption. Our model has linear utility: aggregate consumption is the appropriate welfare measure in this economy, but misses important factors such as consumption inequality. Furthermore, our calibration implies that the workers' outside options are similar to their wages, and so the quantitative difference between the different paths for consumption are small. They however reflect qualitatively what we have seen from the changes in the unemployment rate: worker protection leads to a welfare improvement only when screening does not respond much.

Figure 6 displays the heterogeneous impact of the worker protection policy. We see that its



Response of unemployment rates and consumption by worker type. Dashed lines and continuous lines refer to high-type workers, respectively. Color-coding of lines is dependent on σ_k and as in Figure 5.

Figure 6: Heterogeneous impact of worker protection

impact on high-type workers is negligible: it has the majority of its effect – good or bad – on the unemployment rate and consumption of low-type workers.

Firing costs We have assumed that worker protection prevents firms from laying off individual workers, which is more similar to the current status in Sweden[11]. If instead we assumed a layoff cost, for example in form of a severance payment, the welfare results would depend on the size of the cost.

If the cost is sufficiently high as to prevent layoffs, the outcomes would be identical. If the cost is sufficiently small, firms would still layoff bad matches. This increased risk associated with hiring at random would still impede job openings. However, worker protection no longer improves the quality of the pool of unemployed, as bad matches (and with it, disproportionately low-typed workers) are still being laid off. The worker protection no longer combats the screening externality and has no chance of generating a welfare benefit.

This emphasizes the importance of the exact details when designing worker protection: two policies that may be very similar in spirit can lead to very different outcomes.

[11]Sweden, a neighboring country of Denmark with many similar socioeconomic policies, is a natural candidate for comparison. In Sweden, workers under permanent contract cannot be laid off for individual reasons. The only valid justification for layoff is to reduce the size of an establishment, in which case workers have to be laid off in the decreasing order of tenure. In rare circumstances, individual workers can be laid off in agreement with the unions.

5 Conclusion

We have built a model with heterogeneous workers that differ in their ability to find good matches. Firms can either screen during the hiring process, or learn about the type during employment. Firms screen on match quality. However, match quality is correlated with the worker type: when firms screen, they effectively worsen the quality of the pool of unemployed.

This leads to a screening externality where changes in match productivity – such as an aggregate TFP shock – interact with the firms' screening decision and thereby the quality of the pool of unemployed, amplifying the aggregate effect on market tightness. We have shown that the quantitative effect of this screening externality is large: the qualitative effect of the worker protection policy depends on whether the screening response is sufficiently small.

This screening externality is only present when firms cannot target workers by type: if the labor market was segmented by type, an increase in the unemployment rate of one type does not affect job-finding rates of other types. Yet, we empirically documented that low-typed workers are present in most occupations, industries, age and education groups. This suggests that firms may indeed face difficulties in targeting workers of a specific type in the hiring process, and that the screening externality may be of quantitative significance in business cycle dynamics.

Further research is warranted to establish firmly the extent to which firms can direct their search towards workers by type [12].

[12] The approach highlighted in Lentz and Moen (2017) appears promising for this avenue.

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A Proofs

A.1 Proof of Proposition 1

We drop time indices without loss of generality.

G and g denote the CDF and the PDF of the screening cost variable k. The sign of changes in the screening shares is hence equal to the sign of changes to the screening cutoff $\underline{k} = -(1-P(s_u))J^b$: the higher the screening cutoff $\underline{k}$, the higher the probability to draw a screening cost below $\underline{k}$:

$$\begin{aligned}\frac{\partial G(\overline{k})}{\partial s_u} &= g(\overline{k})\frac{\partial \overline{k}}{\partial s_u}\\ \frac{\partial \overline{k}}{\partial s_u} &= P'(s_u)J^b\\ &= -(\mathcal{P}_h - \mathcal{P}_\ell)J^b > 0.\end{aligned}$$

By assumption, $P_h > P_\ell$: an increase in the share of low types among the unemployed reduces the probability of good matches. Since $J^b < 0$, we have that this expression is negative.

It is straight-forward to see that the cross derivative $[\partial^2\overline{k}]/[\partial s_u \partial(P_h - P_\ell)] = -J^b > 0$: the effect is stronger the larger the heterogeneity between the two types.

As for the second part of the proposition, clearly

$$\frac{\partial \underline{k}}{\partial J^b} = -(1 - P(s_u)) < 0.$$

A.2 Proof of Proposition 2

First, we demonstrate the following property:

Lemma 1

$$\begin{aligned}&Z_h > Z_\ell,\\ &Z_i \equiv \frac{1}{\delta+\hat{\delta}}(1-P_i)(1-S)f + \frac{1}{\delta}P_i f.\end{aligned}$$

We begin by observing that $Z_i > 0$: the expression can be rewritten as

$$P_i f\left[\frac{1}{\delta} - \frac{1}{\delta+\hat{\delta}}(1-S)\right] \geq 0.$$

For $S = 0$, this boils down to $1/\delta > 1/(\delta+\hat{\delta})$, and for $S = 1$ this equals to $1/\delta > 0$. Therefore, Z_i is simply scaled by P_i for the two types. As $P_h > P_\ell$, the proof of the Lemma follows immediately.

Second, we note that it is sufficient to show the following:

Lemma 2 *The share of low types among the unemployed increases if the weighted derivative of the unemployment rate of type ℓ is larger than that of type h: $\frac{\partial u_\ell}{\partial S}\frac{1}{u_l} > \frac{\partial u_h}{\partial S}\frac{1}{u_h} \Rightarrow \frac{\partial s_u}{\partial S} > 0$*

To see this, note that

$$
\begin{aligned}
\frac{\partial s_u}{S} &= \frac{\frac{\partial u_\ell}{\partial S}(u_\ell + u_h) - \frac{\partial u_\ell + u_h}{\partial S} u_\ell}{(u_\ell + u_h)^2} \\
\frac{\partial s_u}{S} &> 0 \Rightarrow \\
\frac{\partial u_\ell}{\partial S}(u_\ell + u_h) &> \frac{\partial u_\ell + u_h}{\partial S} u_\ell \\
\frac{\partial u_\ell}{\partial S}\frac{1}{u_\ell} &> \frac{u_h}{\partial S}\frac{1}{u_h}.
\end{aligned}
$$

Finally, to prove Proposition 2, we begin with the steady state mass of unemployed workers of each type, where we use the shorthand $m = m(\theta)$.

$$
u_i = L_i \frac{1}{1 + \frac{1}{\delta} P_i m + \frac{1}{\delta + \hat{\delta}}(1 - P_i)(1 - S)m}
$$

The derivative of the mass with respect to screening is given by

$$
\begin{aligned}
\frac{\partial u_i}{\partial S} &= L_i \frac{\frac{1}{\delta + \hat{\delta}}(1 - P_i)(1 - S)m}{\left(1 + \frac{1}{\delta} P_i m + \frac{1}{\delta + \hat{\delta}}(1 - P_i)(1 - S)m\right)^2} \\
\frac{\partial u_i}{\partial S}\frac{1}{u_i} &= \frac{\frac{1}{\delta + \hat{\delta}}(1 - P_i)(1 - S)m}{1 + P_i m \left(\frac{1}{\delta} - \frac{1}{\delta + \hat{\delta}}(1 - S)\right) + \frac{1}{\delta + \hat{\delta}}(1 - S)m}
\end{aligned}
$$

Note that X_i is decreasing in P_i. We have $P_h > P_\ell$, and thus $X_h < X_\ell$. Furthermore, per Lemma 1, we have $Z_h > Z_\ell$. The remainder of the denominator is not type specific. Since this expression has both a larger numerator and a smaller denominator for the low type, we have shown that

$$
\frac{\partial u_\ell}{\partial S}\frac{1}{u_\ell} > \frac{\partial u_h}{\partial S}\frac{1}{u_h}.
$$

B Descriptive statistics of the two clusters

In what follows, we characterize the high- and low type workers that are active in the Danish labour market. Figure 7 visualizes the distribution of the two types by age and education. With two exceptions, approx 10% of workers in any given age group are of the low type. First, the lowest age group of workers with on average 25 years are have a disproportionately high share of low-type workers. Presumably, this comes from many workers that do part time work during their education. Second, there are close to no workers in the retirement that are identified as low type. This is likely because only high-type workers in that age group self-select into the labour market.

This figure also shows the distribution of types by education. Less educated workers are more likely to be identified as low-type workers. Yet, once we exclude workers with less than a lower secondary education, the difference in the share of low-type workers by educational group becomes negligible.

Figures 8 and 9 display the type distributions by occupation and industry. We find low-type workers in virtually every occupation and industry. Some occupations (mostly managerial, education and health) and industries (oil, telecommunications, IT, finance, consulting and research) are the host to a disproportionately low number of less than 3% of low-type workers. Some occupations (notably sales and personal services) and industries (transportation, hotels, restaurants, publishing) are outliers in that they contain a large share – up to 20% of low-type workers. The vast majority of occupations and industries host between 85% and 95% of low-type workers.

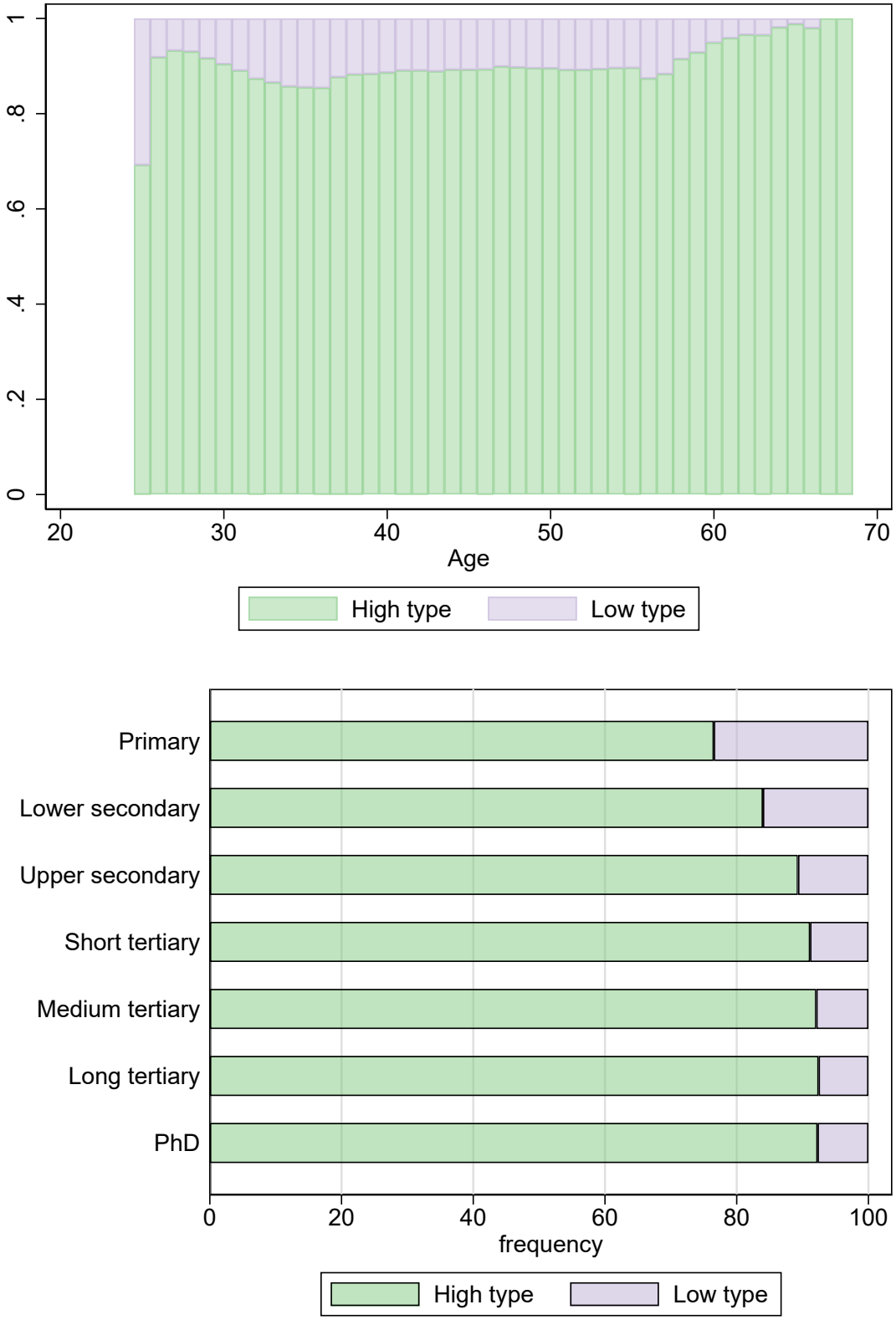

Figure 7: Share of high types by age and education

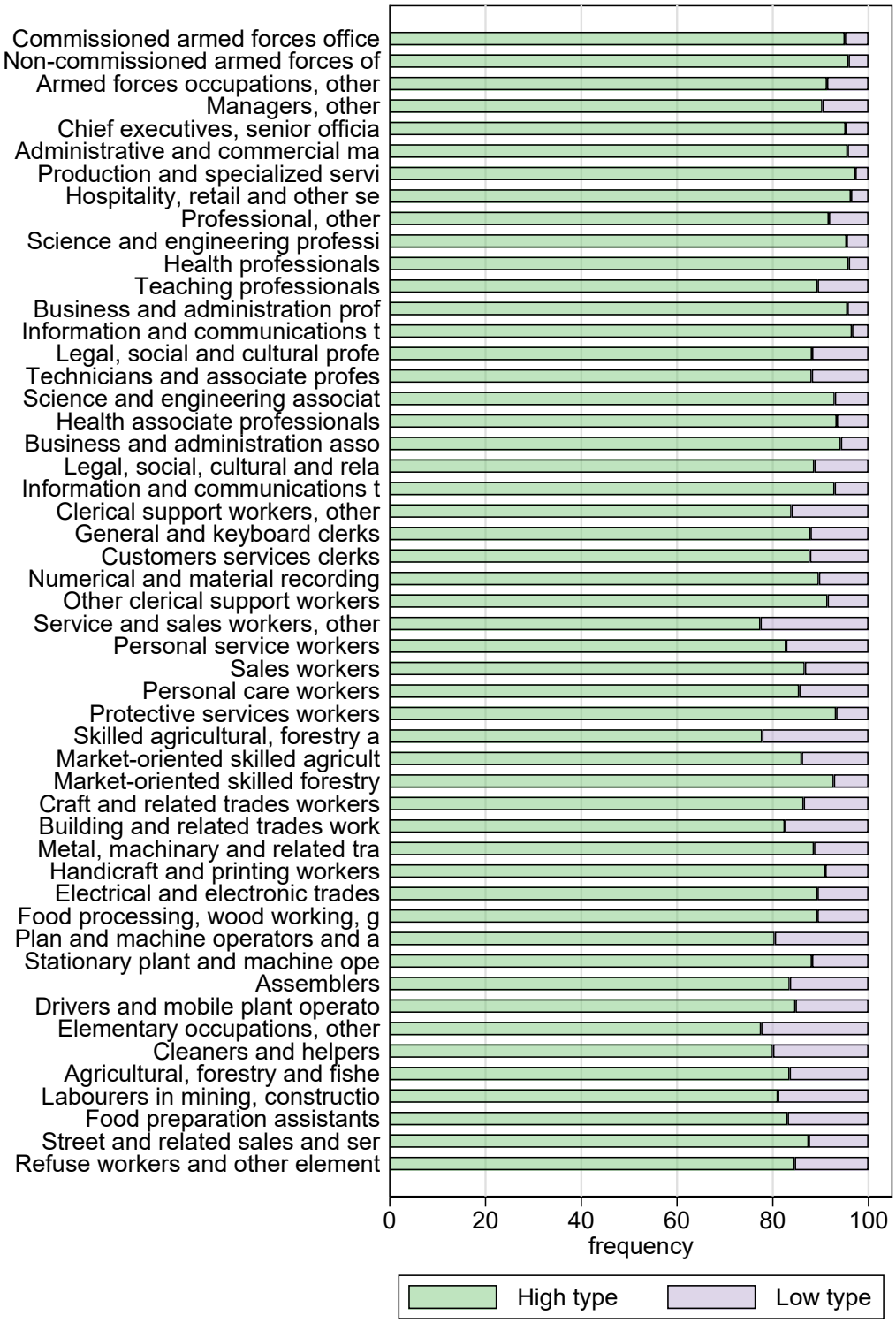

Figure 8: Share of high types by occupation

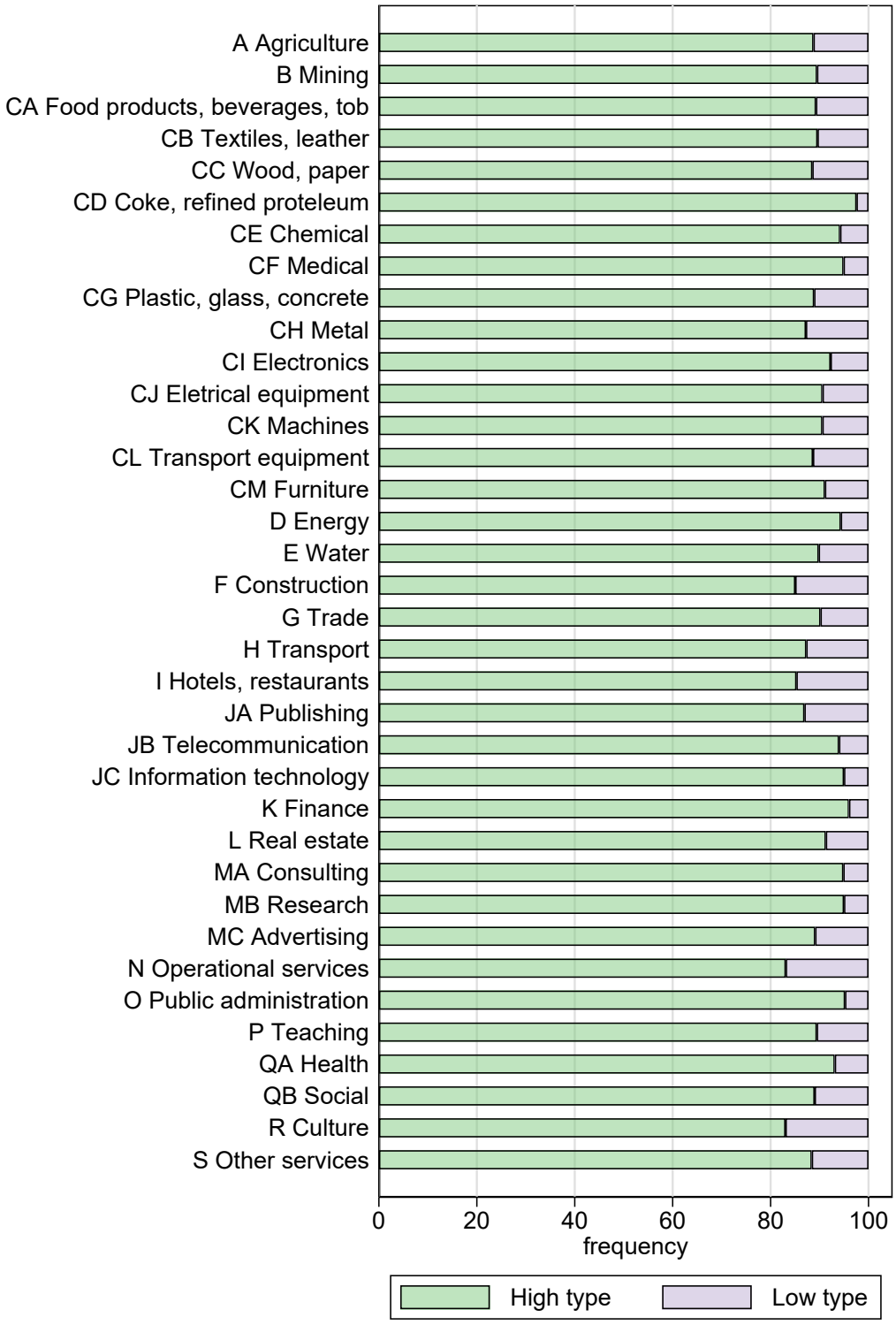

Figure 9: Share of high types by industry

C Clustering workers

We follow Gregory, Menzio, and Wiczer (2021) closely in our approach of clustering workers using the k-means algorithm, and adapt it slightly to our administrative data, which is much richer in the number of workers, but covers a shorter period.

We begin with the matched employer-employee register data, the BFL. The source of this data is transfers from the employer to the employees. We use the payment data to compute the length of individual employer-employee match durations at the monthly level. Some wage payments may happen after a match has ended, for example remaining bonuses or reimbursements: we remove any single employer-employee observation that occurs more then 12 months after the previous one. We treat recall employment as a new match: any break in payments that lasts more than 12 months will lead us to define a new match for the employer-employee pair and reset the duration.

For each match, we compute the length of the preceding unemployment spell. Here, we use the "COMB" definition from the main text, where we leverage additional information on unemployment benefits from DREAM. The take up rate for these benefits is high[13], and we observe that a much larger share of workers is receiving unemployment benefits than classified as nonemployed (compare BFL unemployment and DREAM unemployment in Table 1).

We are primarily interested in workers with a high attachment to the labor force, and so we follow Gregory, Menzio, and Wiczer (2021) in excluding any worker that has a non-employment spell that lasts more than 2 years. Furthermore, we focus on primary-aged workers and remove workers that at any point in our sample are below the age of 30 or above 65.

Let t denote the duration of an employment or unemployment spell in months. For each worker, we compute the following 10 moments:

1. Share of employment spells with $t < 3$
2. Share of employment spells with $3 < t < 12$
3. Share of employment spells with $12 < t < 24$
4. Share of employment spells with $t > 24$
5. Share of unemployment spells with $t < 1$
6. Share of unemployment spells with $1 \leq t < 3$
7. Share of unemployment spells with $3 \leq t < 12$
8. Share of unemployment spells with $t \geq 12$
9. Average number of jobs per month observed
10. Average unemployment rate

We standardize each moment by its standard deviation. We weight the four employment-spell related moments and the four unemployment-spell related moments by one-fourth. All moments are independent of the duration for which we observe a given worker: we will not differentially

[13]The primary requirement is one year of payments into a registered unemployment insurance, and registering for benefits is relatively uncomplicated in Denmark.

assign workers to clusters depending on how many years of observations we have for them. With this data in hand, we ask the k-means clustering algorithm to sort these workers into two clusters. We label the cluster with the lower average unemployment rate as "Cluster 1", which corresponds to the high-type workers in the model.

D An economy with probability-dependent wages

In the main text, we assumed that wages do not depend on the productivity of a given match. Here, we instead bring our model closer to Pries and Rogerson (2005) and assume that wages are a function of the probability of a given match being good: $w = w(\pi)$. In our environment, this probability will be given by the share of low-type workers among the unemployed at the time of hire, $P(s_u(t))$. This implies that we need to keep track of the time at which a worker was hired – this challenges our ability to compute the entire transition of the economy from one steady state to another. Therefore, we focus on solving the steady states before and after the introduction of worker protection for this economy, and will show that these steady states are in line with our findings from the main text.

Let $\mathcal{P}$ take the value of one when worker protection prevents firms from separating bad matches. The steady-state value function of the firm and the wages are given by

$$(\rho + \delta + \hat{\delta})J(\pi) = \begin{cases} \hat{J}(\pi) & \text{if } \mathcal{P} = 1 \\ \max\{\hat{J}(\pi), 0\} & \text{if } \mathcal{P} = 0 \end{cases}$$

$$\hat{J}(\pi) = \pi y^g + (1-\pi)y^b - w(\pi) + \hat{\delta}[\pi J(1) + (1-\pi)J(0)]$$

$$w(\pi) = \beta(\pi J^g + (1-\pi)J^b + \alpha[\pi J(1) + (1-\pi)J(0)]) + (1-\beta)b$$

As in the main text, these wages are consistent with the Generalized Nash Bargaining solution where the match is paused, not separated. During this pause, the firm produces nothing, and the worker consumes b (Kaplan and Menzio, 2014).

In the main text, the only effect from learning match quality was the potential separation of the match, and we could use $\hat{\delta}$ indiscriminately for the rate at which bad matches are separated, and the rate at which bad matches are found out.

Here, learning about the quality of a match can lead to a wage change but not a separation, and so we follow Pries and Rogerson (2005) in denoting the rate at which matches learn about their quality as α[14].

In this context, screening vacancies pay k but only match with good candidates, receiving value $J(1)$. Vacancies that hire at random receive $J(P(s_u))$:

$$\begin{aligned} R^S(k) &= P(s_u)J(1) - k \\ R^N &= J(P(s_u)) \\ V &= SR^S(k) + (1-S)R^N, \end{aligned}$$

where the screening share is given by

$$\begin{aligned} S &= G(\underline{k}) \\ \underline{k} &= P(s_u)J(1) - J(P(s_u)) \end{aligned}$$

The remainder of the model is straight forward and in line with what we have derived in the main text.

[14] As Pries and Rogerson (2005) show, this can be modeled explicitly as optimal learning from a noisy signal.

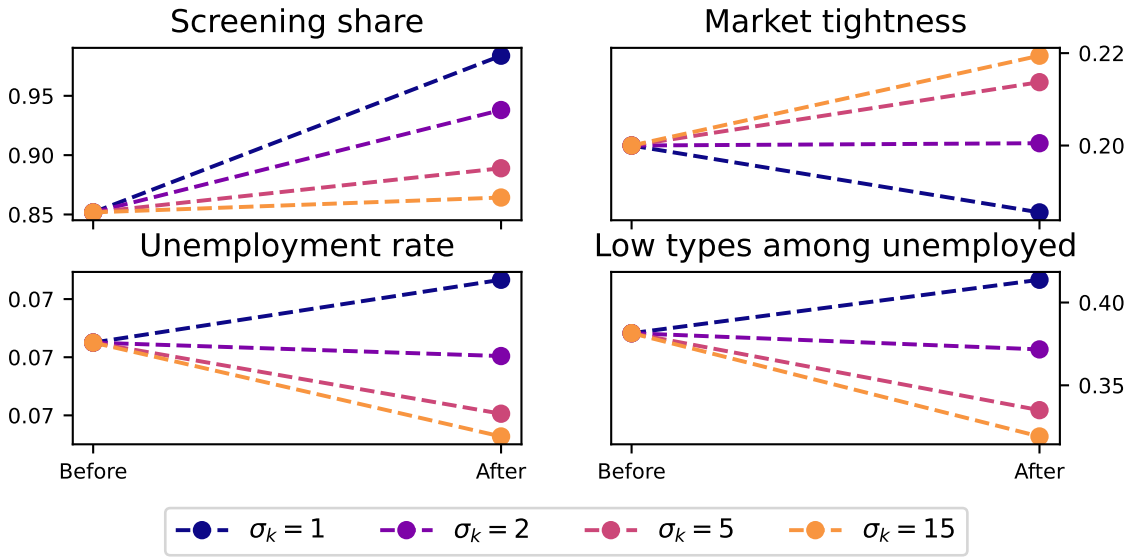

Figure 10: Impact of worker protection in the economy with probability-dependent wages

Informally, a steady state is characterized by $\{e_i^g, e_i^b\}_{i\in\{\ell,h\}}$, θ, S, s_u, and a set of value functions, such that

- Market tightness θ is consistent with free entry
- s_u is consistent with the (un)employment distribution of both types
- The screening share S is consistent with the value functions and s_u
- The value functions are given by our description above.

We calibrate this model according to our procedure in the main text for a variety of σ_k parameters. Figure 10 shows the impact of the introduction of worker protection for a variety of σ_k values. As in the main model, calibrations with a higher value for σ_k feature a smaller response of screening, an increase in market tightness θ, a decrease in the unemployment rate and the share of low-typed workers among the unemployed, s_u. When σ_k is small enough – and the screening response is large enough –, the sign on the changes in u, s_u, θ flips.